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ANALYSIS OF THE TEMPERATURE FIELD OF CYLINDRICAL FLOW BASED ON AN ON-THE-AVERAGE EXACT SOLUTION

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The temperature field in a well is constructed on the basis of an on-the-average exact solution, which allows investigation of problems of subterranean thermodynamics and heat and mass transfer. The problem is represented in the form of a sequence of problems of a mixed type, whose solutions give corresponding asymptotic-expansion coefficients and the form of the remainder term and the functions taking into account the presence of the boundary layer, for which analytical solutions are also found. It is shown that the proposed modified asymptotic method provides vanishing of the solution of the averaged problem for the remainder term.

Key words: *well, temperature field, asymptotic method.*

Introduction. Temperature measurements are widely used to study wells and reservoirs [1–4]. Because of the great complexity of thermodynamic processes, whose description requires the use of mechanical models of multiphase flows in pipes, approximate analytical solutions of the main problem of temperature logging have been obtained only for the temperature averaged over the well cross section. Previously, it has been shown [5] that real radial temperature distributions can be found by asymptotic methods as a first approximation.

Attempts to construct theory of thermal processes in a well by using asymptotic methods were undertaken in [6–10]. In [9], an approximate analytical solution taking into account the radial velocity profile was constructed under the assumption of a constant temperature gradient of the rock surrounding the well. A new method for calculating the temperature averaged over the well cross section is described in [10].

Unlike in [1–10], in the present work, viscous boundaries [12] are eliminated by constructing solutions that take into account the presence of the boundary layer [11]. As an extension of the previously proposed approach [10], solutions of the main problem of temperature logging in the zero and first approximations are found on the basis of an on-the-average exact asymptotic solution.

Mathematical Formulation of the Problem. Figure 1 shows the geometry of the problem of the temperature field of fluid flow in a pipe of radius r_0 It is assumed that the surrounding medium is homogeneous and anisotropic [14] and the temperature of rock away from the well varies linearly with the well depth z_d; the range of depths is considered in which there is no effect of seasonal temperature fluctuations on the surfaces. The required solution is subject to a symmetry condition which specifies that the derivative with respect to the radial z_d axis of cylindrical coordinates directed upward along the axis of the pipe vanishes at the center of the well.

The fluid velocity field in the pipe has only one nonzero component — in the z_d direction: $\boldsymbol{v} = (0, 0, v)$. It is assumed that the fluid velocity in the pipe does not depend on the distance to the axis of the well and coincides with the averaged value. The moving fluid also acquires fictitious orthotropic properties due to the turbulence effect [13].

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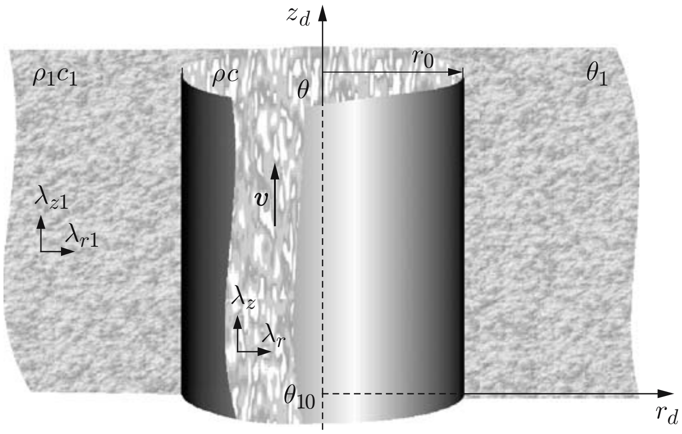

Fig. 1. Geometry of the problem.

We introduce the following dimensionless variables:

$$r = \frac{r_d}{r_0}, \quad z = \frac{z_d}{D}, \quad \mathrm{Fo} = \frac{\tau a_{1r}}{r_0^2}, \quad T_1 = \frac{\theta_1 - \theta_{01} + \Gamma z_d}{\theta_0}, \quad \theta_0 = \Gamma D, \quad \chi = \frac{c_1 \rho_1}{c\rho},$$

$$\nu = \frac{r_0}{D}, \quad T = \frac{\theta - \theta_{01} + \Gamma z_d}{\theta_0}, \quad \Lambda = \frac{\lambda_{1r}}{\lambda_r}, \quad \mathrm{Pe} = \frac{v r_0}{a_{1r}}, \quad H = \frac{\eta \rho g r_0}{\nu \theta_0}.$$

Then, the mathematical formulation of the problem includes the thermal-conductivity equation for the rock mass surrounding the pipe

$$\rho_1 c_1 \frac{\partial \theta_1}{\partial \tau} = \lambda_{1z} \frac{\partial^2 \theta_1}{\partial z^2} + \lambda_{1r} \frac{1}{r} \frac{\partial}{\partial r}\Big(r \frac{\partial \theta_1}{\partial r}\Big), \qquad r > r_0, \quad \tau > 0, \quad z > 0 \tag{1}$$

and the equation of convective thermal conductivity of the fluid in the well

$$\rho c \frac{\partial \theta}{\partial \tau} = \lambda_z \frac{\partial^2 \theta}{\partial z^2} + \lambda_r \frac{1}{r} \frac{\partial}{\partial r}\Big(r \frac{\partial \theta}{\partial r}\Big) - \rho c v \frac{\partial \theta}{\partial z} + q, \qquad r < r_0, \quad \tau > 0, \quad z > 0 \tag{2}$$

(λ_r and λ_z are the components of the thermal conductivity tensor of the fluid corresponding to the axes). On the boundary between the pipe and the surrounding rock mass, we specify the conditions of equality of temperatures and heat fluxes:

$$\theta\Big|_{r=1} = \theta_1\Big|_{r=1}, \qquad \lambda_r \frac{\partial \theta}{\partial r}\Big|_{r=1} = \lambda_{1r} \frac{\partial \theta_1}{\partial r}\Big|_{r=1}. \tag{3}$$

The initial conditions correspond to the natural unperturbed temperature of the Earth, which increases with depth z under the linear law

$$\theta\Big|_{\tau=0} = \theta_1\Big|_{\tau=0} = \theta_{01} - \Gamma z \tag{4}$$

and is equal to the temperature at the points of the surrounding rock mass remote from the pipe

$$\theta_1\Big|_{r\to\infty} = \theta_{01} - \Gamma z. \tag{5}$$

At the point $z = 0$, the flow temperature is known:

$$\theta\Big|_{z=0} = \theta_{10}(\tau). \tag{6}$$

For the solution of the problem to be unique, it is necessary to add boundary conditions on the z axis, but it is not necessary to write them in explicit form because the second derivatives with respect to z can be ignored, as shown below.

In Eqs. (1) and (2), the terms containing the second derivative of the temperature with respect to the vertical coordinate there appears the small multiplier — the square of $\nu = r_0/D \approx 10^{-4}$ ($r_0 \approx 0.1$ m is the radius of the well and $D \approx 10^3$ m is the length of the well or the interval in which the temperature is measured). Therefore, the terms containing the coefficient ν^2 in Eqs. (1) and (2) are omitted.

In the problem in question written in dimensionless variables, the asymptotic-expansion parameter ε formally introduced by replacing Λ with $\varepsilon\Lambda$ [5]:

$$\frac{\partial T_1}{\partial\,\mathrm{Fo}} - \frac{1}{r}\frac{\partial}{\partial r}\Big(r\,\frac{\partial T_1}{\partial r}\Big) = 0, \qquad r > 1, \quad \mathrm{Fo} > 0, \quad z > 0,$$

$$\frac{\partial T}{\partial\,\mathrm{Fo}} - \frac{\chi}{\varepsilon\Lambda}\,\frac{1}{r}\frac{\partial}{\partial r}\Big(r\,\frac{\partial T}{\partial r}\Big) + \mathrm{Pe}\,\nu\Big(\frac{\partial T}{\partial z} - 1 + H\Big) - Q(r, z, \mathrm{Fo}) = 0,$$

$$r < 1, \qquad \mathrm{Fo} > 0, \qquad z > 0, \tag{7}$$

$$T\Big|_{r=1} = T_1\Big|_{r=1}, \qquad \frac{\partial T}{\partial r}\Big|_{r=1} = \varepsilon\Lambda\,\frac{\partial T_1}{\partial r}\Big|_{r=1},$$

$$T\Big|_{\mathrm{Fo}=0} = T_1\Big|_{\mathrm{Fo}=0} = 0, \qquad T_1\Big|_{r\to\infty} = 0, \qquad T\Big|_{z=0} = T_0(\mathrm{Fo}).$$

Here $T_0(\mathrm{Fo})$ is the reservoir temperature signal which is determined as the relative difference in temperature between the well $\theta_{10}(\tau)$ and the remote medium θ_{01} at $z = 0$: $T_0(\mathrm{Fo}) = (\theta_{10}(\tau) - \theta_{01})/\theta_0$.

The solution of problem (7) is represented as the following formulas which are asymptotic for the parameter ε

$$T = T^{(0)} + \sum_{i=1}^{n} \varepsilon^i T^{(i)} + \Theta^{(i)}, \qquad T_1 = T_1^{(0)} + \sum_{i=1}^{n} \varepsilon^i T_1^{(i)} + \Theta_1^{(i)}, \tag{8}$$

where i is the approximation number. The method of solution is described in [9].

The zero approximation represents the temperature averaged over the well cross section, which is of significance for estimation of physical processes.

The accuracy of the first approximation is determined by the estimate of the remainder term. The coefficient $T^{(1)}$ can be found in such a manner that the averaged value of the remainder term vanishes ($\langle\Theta\rangle = 0$) for any values of the parameter ε. This asymptotic approximation is referred to as an on-the-average exact one.

On the space of Laplace–Carson images, the solution of the problem of the well temperature field in a zero approximation has the form

$$T^{(0)u} = \int_0^z \frac{\mathrm{Pe}\,\nu(1 - H) + 2Q_1^u(1, \xi, p)}{\mathrm{Pe}\,\nu}\,\mathrm{e}^{-\alpha_2(z-\xi)}\,d\xi + T_0^u(p)\,\mathrm{e}^{-\alpha_2 z}, \qquad r < 1, \qquad z > 0.$$

In the absence of sources, the expression for the zero coefficient in the space of originals at small times becomes

$$T^{(0)} = T_0\,\mathrm{erfc}\Big(\frac{\chi z}{\mathrm{Pe}\,\nu\sqrt{\mathrm{Fo} - z/(\mathrm{Pe}\,\nu)}}\Big)\Phi\Big(\mathrm{Fo} - \frac{z}{\mathrm{Pe}\,\nu}\Big)$$

$$+ (1 - H)\int_0^z \mathrm{erfc}\Big(\frac{\chi(z - \xi)}{\mathrm{Pe}\,\nu\sqrt{\mathrm{Fo} - (z - \xi)/(\mathrm{Pe}\,\nu)}}\Big)\Phi\Big(\mathrm{Fo} - \frac{z - \xi}{\mathrm{Pe}\,\nu}\Big)\,d\xi, \tag{9}$$

$$\mathrm{Fo} > 0, \qquad r < 1, \qquad z > 0.$$

The obtained solution allows one to construct curves of temperature versus time and vertical coordinate in the zone of influence of reservoir temperature signals (points I). In this zone ($0 < z < 1$), the geothermal temperature gradient ΓD is comparable to the value of the temperature anomaly in the layer θ_{01}. The calculations were performed for the following parameter values: $H = 0.1$, $\mathrm{Pe} = 10^4$, and $\nu = 10^{-4}$.

Figure 2 shows curves of the relative temperature in the well versus dimensionless vertical coordinate in the zone of influence of reservoir temperature signals at various times. For small values of z, the curves constructed with and without taking into account the reservoir temperature signal differ significantly. The difference between these curves decreases with increasing z. The difference between the values of $T^{(0)}$ at the points on the curves corresponds to the contribution of the reservoir temperature signal to the total temperature value. An analysis of

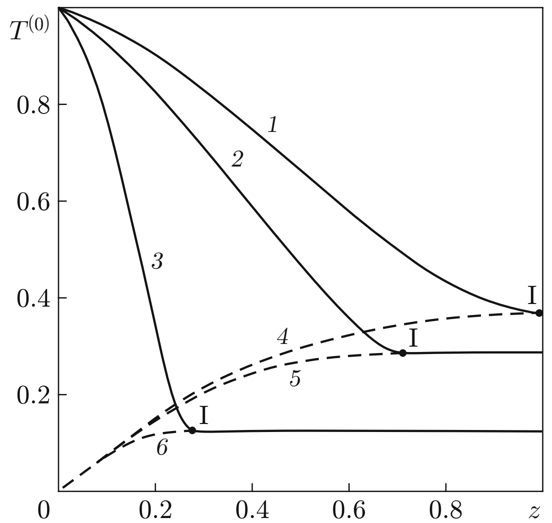

Fig. 2

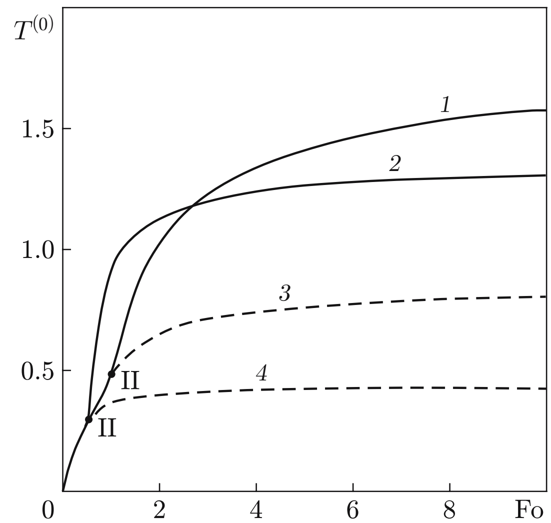

Fig. 3

Fig. 2. Well temperature versus dimensionless vertical coordinate in the zone of influence of reservoir temperature signals at various times: Fo = 1.5 (1 and 4), 1 (2 and 5), 0.3 (3 and 6); dashed curves correspond to a single temperature signal, and solid curves to zero temperature signal.

Fig. 3. Well temperature versus dimensionless time in the zone of influence of reservoir temperature signals at $z = 1$ (1 and 3) and 0.5 (2 and 4); solid curves correspond to a single temperature signal and dashed curves to zero temperature signal.

the curves shows that, as the time increases, the extent of the zone of influence of the reservoir temperature signal increases and is $z = 0.25$ for Fo = 0.3, $z = 0.75$ for Fo = 1, and $z = 1$ for Fo = 1.5.

Figure 3 gives curves of the well temperature in a zero approximation versus dimensionless time in the zone of influence of reservoir temperature signals. From Fig. 3, it follows that, at small times, there is no contribution of reservoir temperature signals; therefore, the corresponding curves 2, 4 and 1, 3 coincide, and with the arrival of the temperature signal (points II), the curves diverge. The difference between the values of $T^{(0)}$ at the points on the curves corresponds to the contribution of the reservoir temperature signal. In Fig. 3, it is evident that the time of arrival of the temperature signal increases with increasing z.

From Eq. (9), it follows that the solution of the problem in the zero approximation describes the time dependence of the temperature averaged over the well cross section but does not describe the radial temperature distribution in the well. To solve the problem of radial temperature distribution, we solve the boundary-value problem for the first coefficients of the expansion [9]. It is obvious that the condition $T^{(1)}\Big|_{z=0} = 0$ cannot be satisfied for any r, which indicates the presence of a boundary layer for small z and, hence the need to change the boundary condition. It is obvious that the boundary condition should provide an increase in the solution accuracy dependent on the value of the remainder term of the asymptotic expansion.

Derivation of an Additional Condition from the Problem for the Remainder Term. Substituting the asymptotic formulas

$$T = T^{(0)} + \varepsilon T^{(1)} + \Theta^{(1)}, \qquad T_1 = T_1^{(0)} + \varepsilon T_1^{(1)} + \Theta^{(1)}$$

into (7) and taking into account that the zero coefficients of the expansion and the radial derivatives of the first coefficients satisfy the problems considered above, we obtain the following problem for the remainder terms of the asymptotic expansions

$$\frac{\partial \Theta_1}{\partial\,\mathrm{Fo}} - \frac{1}{r}\frac{\partial}{\partial r}\Big(r\,\frac{\partial \Theta_1}{\partial r}\Big) = 0,$$

$$\varepsilon\,\frac{\partial \Theta}{\partial\,\mathrm{Fo}} - \frac{\chi}{\Lambda}\,\frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{\partial \Theta}{\partial r}\Big) + \mathrm{Pe}\,\nu\varepsilon\,\frac{\partial \Theta}{\partial z} = -\varepsilon^2\Big(\frac{\partial T^{(1)}}{\partial\,\mathrm{Fo}} + \mathrm{Pe}\,\nu\,\frac{\partial T^{(1)}}{\partial z}\Big),$$

$$\Theta\Big|_{r=1} = \Theta_1\Big|_{r=1}, \qquad \frac{\partial \Theta}{\partial r}\Big|_{r=1} = \Lambda\varepsilon \, \frac{\partial\, (\varepsilon T_1^{(1)} + \Theta_1)}{\partial r}\Big|_{r=1}, \tag{10}$$

$$(\varepsilon T + \Theta)\Big|_{\mathrm{Fo}=0} = 0, \qquad (\varepsilon T_1 + \Theta_1)\Big|_{\mathrm{Fo}=0} = 0,$$

$$(\varepsilon T^{(1)} + \Theta)\Big|_{z=0} = 0, \qquad \Theta_1\Big|_{r\to\infty} = 0.$$

We average problem (10) over r from 0 to 1: $\langle\Theta\rangle = 2\int\limits_0^1 r\Theta\, dr$. If the average integral values of the temperature at the points $z = 0$ and $\mathrm{Fo} = 0$ vanish, i.e.,

$$\langle T^{(1)}\rangle\Big|_{z=0} = 0, \qquad \langle T^{(1)}\rangle\Big|_{\mathrm{Fo}=0} = 0, \tag{11}$$

the averaged problem for the remainder term has a trivial solution, i.e., the solution of the initial problem (7) is exact on the average.

In view of condition (11) in the space of images, the first coefficient of the asymptotic expansion of the well temperature is given by the formula

$$T^{(1)u} = -\frac{\Lambda r^2}{2}\sqrt{p}\, kT^{(0)u} + \frac{\Lambda}{4}\sqrt{p}\, kT_0^u(p)\, \mathrm{e}^{-\alpha_2 z}$$
$$+\Lambda\int\limits_0^z \mathrm{e}^{-\alpha_2(z-\xi)}\Big(\frac{\chi p k^2}{2\,\mathrm{Pe}\,\nu}\, T^{(0)u} + \frac{\sqrt{p}\, k(1-H)}{4}\Big) d\xi + \frac{\Lambda}{\chi}\Big(2Q_3^u(1,0,p) - \frac{Q_1^u(1,0,p)}{4}\Big)\mathrm{e}^{-\alpha_2 z}$$
$$+\frac{\Lambda r^2}{2\chi}\, Q_1^u(1,z,p) + \frac{2\Lambda k\sqrt{p}}{\mathrm{Pe}\,\nu}\int\limits_0^z Q_2^u(1,\xi,p)\, \mathrm{e}^{-\alpha_2(z-\xi)}\, d\xi$$
$$-\frac{\Lambda}{\chi}\, Q_2^u(r,z,p) - \frac{\Lambda}{4\chi}\int\limits_0^z \Big(\alpha_2 Q_1^u(1,\xi,p) + \frac{\partial Q_1^u(1,\xi,p)}{\partial \xi}\Big)\mathrm{e}^{-\alpha_2(z-\xi)}\, d\xi$$
$$+2\,\frac{\Lambda}{\chi}\int\limits_0^z \Big(\frac{p}{\mathrm{Pe}\,\nu}\, Q_3^u(1,\xi,p) + \frac{\partial Q_3^u(1,\xi,p)}{\partial \xi}\Big)\mathrm{e}^{-\alpha_2(z-\xi)}\, d\xi, \qquad r<1, \quad z>0. \tag{12}$$

In the absence of sources, the first coefficient of the expansion in the space of originals for small times Fo is defined as follows:

$$T^{(1)} = \Lambda\Phi\Big(\mathrm{Fo} - \frac{z}{\mathrm{Pe}\,\nu}\Big)\Big\{\frac{1-2r^2}{4}\, \frac{T_0}{\sqrt{\pi(\mathrm{Fo} - z/(\mathrm{Pe}\,\nu))}} \exp\Big(-\frac{\chi^2 z^2}{\mathrm{Pe}^2\,\nu^2(\mathrm{Fo} - z/(\mathrm{Pe}\,\nu))}\Big)$$
$$-\frac{\chi(1-H)}{2}\, z\exp(4\chi^2\,\mathrm{Fo})\,\mathrm{erfc}\Big[\chi\Big(2\sqrt{\mathrm{Fo} - \frac{z}{\mathrm{Pe}\,\nu}} + \frac{z}{\mathrm{Pe}\, v\sqrt{\mathrm{Fo} - z/(\mathrm{Pe}\,\nu)}}\Big)\Big]$$
$$+\frac{\chi T_0 z}{2\,\mathrm{Pe}\,\nu}\,\psi\Big(\frac{2\chi z}{\mathrm{Pe}\,\nu}, \mathrm{Fo} - \frac{z}{\mathrm{Pe}\,\nu}\Big)\Big\} + \, + \Lambda\,\frac{1-2r^2}{4}\,(1-H)\int\limits_0^z \Phi\Big(\mathrm{Fo} - \frac{z-\xi}{\mathrm{Pe}\,\nu}\Big)\frac{1}{\sqrt{\pi(\mathrm{Fo} - (z-\xi)/(\mathrm{Pe}\,\nu))}}$$
$$\times\exp\Big(-\frac{\chi^2(z-\xi)^2}{\mathrm{Pe}^2\,\nu^2(\mathrm{Fo} - (z-\xi)/(\mathrm{Pe}\,\nu))}\Big)d\xi + \Lambda\,\frac{\chi(1-H)}{2}\int\limits_0^z \Phi\Big(\mathrm{Fo} - \frac{z-\xi}{\mathrm{Pe}\,\nu}\Big)\exp(4\chi^2\,\mathrm{Fo})$$
$$\times\,\mathrm{erfc}\Big[\chi\Big(2\sqrt{\mathrm{Fo} - \frac{z-\xi}{\mathrm{Pe}\,\nu}} + \frac{z-\xi}{\mathrm{Pe}\,\nu\sqrt{\mathrm{Fo} - (z-\xi)/(\mathrm{Pe}\,\nu)}}\Big)\Big]\, d\xi, \qquad r<1, \quad z>0.$$

The solution found in the first approximation is necessary for a detailed description of the temperature field in the well. This solution allows to one perform calculations of radial temperature distributions in wells and temperature variations with depth.

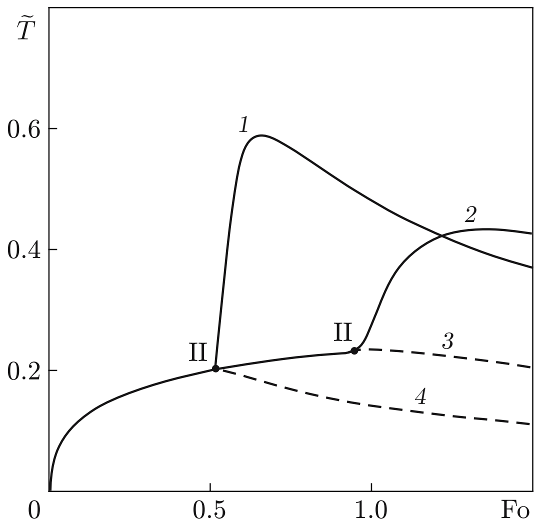

Fig. 4

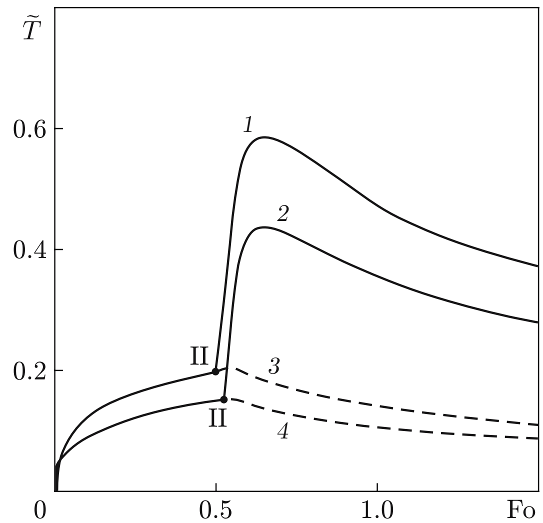

Fig. 5

Fig. 4. Temperature gradient between the well wall and axis versus dimensionless time for various distances from the reservoir: $z = 0.5$ (1 and 4) and 0.9 (2 and 3); solid curves correspond to a single temperature signal and dashed curves to zero temperature signal.

Fig. 5. Temperature gradient between the well wall and axis versus dimensionless time for various dimensionless distances from the well axis: $r = 0$ (1 and 4), 0.5 (2 and 3); solid curves refer to a single temperature signal and dashed curves to zero temperature signal.

Figure 4 shows the temperature gradient between the well wall and axis $\tilde{T} = T^{(1)} - T^{(1)}\Big|_{r=1}$ versus dimensionless time for various distances from the reservoir. From Fig. 4, it follows that the curves have a common feature consisting of the formation of a temperature gradient and its subsequent reduction due to the heating of the rock surrounding the well, resulting in a decrease in the heat flux from the well to the surrounding medium. From an analysis of the curves in Fig. 4, it follows that the time of attainment of the temperature maximum and the value of the relative temperature gradient increase with increasing distance from the reservoir. Away from the well axis, the temperature gradient naturally decreases whereas the time of attainment of the maximum remains approximately the same. Figure 5 gives a curve of the relative temperature gradient between the well wall and axis versus dimensionless time for various dimensionless distances from the well axis. From Fig. 5, it follows that with distance from the well axis, the temperature gradient decreases and the time of attainment of the maximum remains almost unchanged. A common feature in the behavior of curves 1 and 3 and 2 and 4 is that at a certain time, the temperature growth rate increases significantly, which corresponds to the arrival of the reservoir temperature signal. Subsequently, the temperature reaches a maximum and then decreases.

Solution of the Boundary-Layer Problem. We represent the solution of problem (7) in the form of the sums

$$\hat{T} = T + \Pi_z + \Pi_{\mathrm{Fo}}, \qquad \hat{T}_1 = T_1 + \Pi_{z1} + \Pi_{\mathrm{Fo}\,1},$$

where $T = T(r, z, \mathrm{Fo})$ is the regular part of the solution which corresponds to the first approximation found above, $\Pi_z(r, \zeta, \mathrm{Fo})$ and $\Pi_{\mathrm{Fo}}(r, z, \tau)$ are corrections in the expansion in the asymptotic parameter which take into account the presence of the boundary layer, and $\zeta = z/\varepsilon$ and $\tau = \mathrm{Fo}/\varepsilon$ are stretched variables. As a result, we obtain the following problems for the functions Π_z, Π_{Fo}:

$$\frac{\partial \Pi_{z1}}{\partial\,\mathrm{Fo}} = \frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{\partial \Pi_{z1}}{\partial r}\Big),$$

$$\frac{\partial \Pi_z}{\partial\,\mathrm{Fo}} = \frac{\chi}{\Lambda\varepsilon}\,\frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{\partial \Pi_z}{\partial r}\Big) - \frac{\mathrm{Pe}\,\nu R(r)}{\varepsilon}\,\frac{\partial \Pi_z}{\partial \zeta},$$

$$\Pi_z\Big|_{r=1} = \Pi_{z1}\Big|_{r=1}, \qquad \frac{\partial \Pi_z}{\partial r}\Big|_{r=1} = \varepsilon\Lambda\,\frac{\partial \Pi_{z1}}{\partial r}\Big|_{r=1},$$

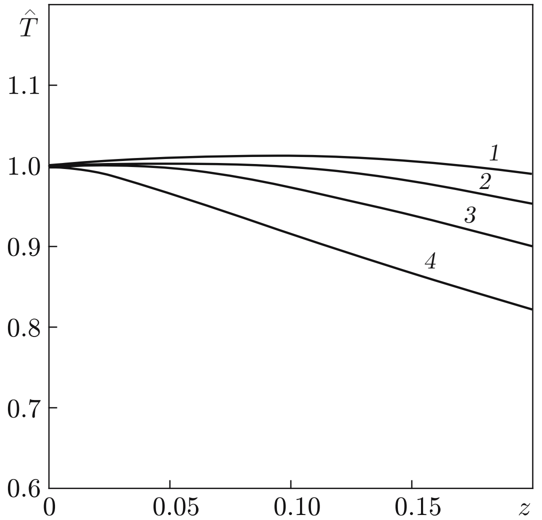

Fig. 6

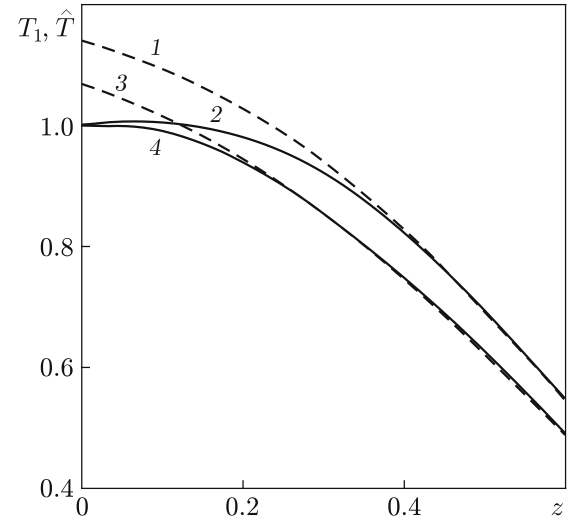

Fig. 7

Fig. 6. Fluid temperature versus dimensionless vertical coordinate taking into account the presence of the boundary layer at various dimensionless distances from the well axis: $r = 0$ (1), 0.4 (2), 0.6 (3), and 0.8 (4).

Fig. 7. Fluid temperature versus dimensionless vertical coordinate without taking into account (1 and 3) and taking into account (2 and 4) the presence of the boundary layer at various dimensionless distances from the well axis: $r = 0$ (1 and 2) and 0.5 (3 and 4).

$$\Pi_{z1}\Big|_{r\to\infty} = 0, \qquad \Pi_z\Big|_{\text{Fo}=0} = 0, \qquad \Pi_{z1}\Big|_{\text{Fo}=0} = 0,$$

$$\Pi_z\Big|_{\zeta=0} = -T\Big|_{z=0}, \qquad \Pi_{z1}\Big|_{\zeta=0} = -T_1\Big|_{z=0}$$

in the vicinity of $z = 0$ and

$$\frac{1}{\varepsilon}\,\frac{\partial \Pi_{\text{Fo}\,1}}{\partial \tau} = \frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{\partial \Pi_{\text{Fo}\,1}}{\partial r}\Big),$$

$$\frac{\partial \Pi_{\text{Fo}}}{\partial \tau} = \frac{\chi}{\Lambda}\,\frac{1}{r}\,\frac{\partial}{\partial r}\Big(r\,\frac{\partial \Pi_{\text{Fo}}}{\partial r}\Big) - \varepsilon\,\text{Pe}\,v R(r)\,\frac{\partial \Pi_{\text{Fo}}}{\partial z},$$

$$\Pi_{\text{Fo}}\Big|_{r=1} = \Pi_{\text{Fo}\,1}\Big|_{r=1}, \qquad \frac{\partial \Pi_{\text{Fo}}}{\partial r}\Big|_{r=1} = \varepsilon\Lambda\,\frac{\partial \Pi_{\text{Fo}\,1}}{\partial r}\Big|_{r=1},$$

$$\Pi_{\text{Fo}}\Big|_{\tau=0} = -T\Big|_{\text{Fo}=0}, \qquad \Pi_{\text{Fo}\,1}\Big|_{\tau=0} = -T_1\Big|_{\text{Fo}=0},$$

$$\Pi_{\text{Fo}}\Big|_{z=0} = 0, \qquad \Pi_{\text{Fo}\,1}\Big|_{r\to\infty} = 0$$

in the vicinity of $\text{Fo} = 0$. Solutions of these problems are obtained by the method of separation of variables and are expressed in terms of Bessel functions of the real argument $J_0(\mu_n r)$:

$$\Pi_z^{(1)} = -2\sum_{n=1}^{\infty}\frac{J_0(\mu_n r)}{J_0^2(\mu_n)}\exp\Big(-\mu_n^2\,\frac{\chi}{\varepsilon\Lambda\,\text{Pe}\,\nu}\,z\Big)\int\limits_0^1 rT^{(1)}\Big|_{z\to 0}J_0(\mu_n r)\,dr,$$

$$\Pi_{\text{Fo}}^{(1)} = -2\sum_{n=1}^{\infty}\frac{J_0(\mu_n r)}{J_0^2(\mu_n)}\exp\Big(-\mu_n^2\,\frac{\chi}{\varepsilon\Lambda}\,\text{Fo}\Big)\int\limits_0^1 rT^{(1)}\Big|_{\text{Fo}\to 0}J_0(\mu_n r)\,dr$$

[μ_n are the roots of the equation $J_1(\mu) = 0$]. The values of $T^{(1)}\Big|_{\text{Fo}\to 0}$, $T^{(1)}\Big|_{z\to 0}$ are determined according (12). It is easy to show that in the absence of sources, $T^{(1)}\Big|_{\text{Fo}\to 0} = 0$. This implies that, in the vicinity of $\text{Fo} = 0$ at $Q = 0$, the boundary layer disappears.

Figure 6 shows a curve of the fluid temperature versus depth z at various distances from the well axis with the presence of the boundary layer taken into account. It is evident that the fluid is heated more strongly in approaching the well axis. (This conclusion agrees with physical considerations.) All curves satisfy the boundary condition $T|_{z=0} = 1$, which is due to the presence of the boundary layer. Figure 7 gives temperature curves without taking into account (curves 1 and 3) and taking into account the presence of the boundary layer (curves 2 and 4). From Fig. 7, it follows that, if the boundary layer is taken into account, the results of the temperature calculations correspond to the conditions of the problem $T|_{z=0} = 1$. Curves 1 and 3, plotted ignoring the boundary layer, do not satisfy this condition, but it is necessary to note that the difference between the calculated results does not exceed 17% at $r = 0$ and decreases with increasing z. Coincidence of the curves constructed with and without taking into account the boundary layer is observed at $z > 0.35$ ($r = 0$) and $z > 0.2$ ($r = 0.5$).

Thus, in the present paper, we constructed an on-the-average exact solution in the form $T = T^{(0)} + \varepsilon(T^{(1)} + \Pi^{(1)})$, where the zero approximation describes the temperature values averaged over the well cross section, the first approximation allows one to study radial temperature distributions in the case of no boundary layer, and the solution in the region of viscous boundaries is refined by means of a function which takes into account the presence of the boundary layer.

Conclusions. The concept of on-the-average exact solution allows one to obtain analytical solutions of the problem of the well temperature field taking into account heat exchange with the rock surrounding the well for an arbitrary vertical fluid temperature gradient and to estimate the contribution of the temperature signal of the perforated reservoir to the well temperature field.

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